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INTEGER LINEAR PROGRAMMING MODEL FOR INVESTIGATING NEAR-INTERVAL EDGE-COLORINGS OF COMPLETE BIPARTITE GRAPHS

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ABSTRACT

A proper edge-coloring of a graph G is a mapping $\alpha: E(G) \to \mathbb{N}$ such that $\alpha(e) \neq \alpha(e')$ for every pair of adjacent edges e and e' in G. If α is a proper edge-coloring of G and $v \in V(G)$, then $S_G(v, \alpha)$ denotes the set of colors appearing on edges incident to v. A proper edge-coloring α of a graph G with colors $1, \dots, t$ is called a near-interval t-coloring if all colors are used, and for each vertex $v \in V(G)$, $S_G(v, \alpha)$ is an interval of integers with no more than one gap. If a graph G has such a coloring, the minimum number of colors in a near-interval coloring of a graph G is denoted by $w^1(G)$. It is known that all complete bipartite graphs admit near-interval colorings. In this paper, we propose an integer linear programming (ILP) model to determine or bound the parameter $w^1(K_{m,n})$ $(m, n \in \mathbb{N})$ for complete bipartite graphs.

Keywords: proper edge-coloring, near-interval coloring, interval coloring, complete bipartite graph, integer linear programming.

Introduction

We use [1, 2] for terminology and notation not defined here. All graphs considered are finite, undirected, and contain no loops or multiple edges. For a graph G, $V(G)$ and $E(G)$ denote the sets of vertices and edges of G, respectively. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$, the maximum degree of G by $\Delta(G)$, the chromatic index of G by $\chi'(G)$, and the diameter of G by $diam(G)$.

A proper edge-coloring of a graph G is a mapping $\alpha: E(G) \to \mathbb{N}$ such that $\alpha(e) \neq \alpha(e')$ for every pair of adjacent edges e and e' in G. If α is a proper edge-coloring of G and $v \in V(G)$, then $S_G(v, \alpha)$ (or $S(v, \alpha)$) denotes the set of colors appearing on edges incident to v. The smallest and largest colors of $S(v, \alpha)$ are denoted by $\underline{S}(v, \alpha)$ and $\overline{S}(v, \alpha)$, respectively. A near-interval t-coloring (or interval

$(t, 1)$-coloring) [3, 4] of a graph G is a proper edge-coloring $\alpha: E(G) \to \{1, \dots, t\}$ such that all colors are used and, for every vertex $v \in V(G)$,

$$d_G(v) - 1 \leq \overline{S}(v, \alpha) - \underline{S}(v, \alpha) \leq d_G(v).$$

Equivalently, $S_G(v, \alpha)$ is an interval of integers with no more than one gap. A graph G is near-interval colorable if it has a near-interval t-coloring for some $t \in \mathbb{N}$. The set of all near-interval colorable graphs is denoted by $\mathfrak{N}^1$. For a graph $G \in \mathfrak{N}^1$, the minimum number of colors in a near-interval coloring of G is denoted by $w^1(G)$.

Near-interval colorings were introduced by Petrosyan and Arakelyan [4] as a natural generalization of interval edge-colorings [5, 6]. Upper bounds on the number of colors in near-interval colorings in terms of $\Delta(G)$, $diam(G)$, and $|V(G)|$ were obtained in [3, 4]. Recently, Casselgren, Małafiejski, Pastuszak and Petrosyan [7] proved that for subcubic graphs G the problem of determining the exact value of $w^1(G)$ is NP-complete. They also showed that all complete bipartite graphs belong to $\mathfrak{N}^1$ and initiated the determination of $w^1(K_{m,n})$ as a natural open problem. For complete bipartite graphs, some upper bounds and exact values were obtained in [8].

A complementary line of work uses integer linear programming (ILP) as a computational tool for interval edge-coloring problems. In [9] and [10], ILP formulations and modern solvers were used to study interval colorings and minimum-deficiency problems, showing that well-designed models can produce strong bounds together with certificates of optimality and can therefore serve as an effective aid in conjecture testing. Motivated by these developments, we formulate an ILP model adapted to near-interval colorings of complete bipartite graphs and use it to compute $w^1(K_{m,n})$ for a range of small instances, providing explicit colorings and additional evidence for the structure suggested by existing results.

Integer Linear Programming Model

Our computational approach is guided by integer programming formulations for minimum-deficiency interval coloring developed in [9]. That work discusses four closely related modeling perspectives. In the complete bipartite setting considered here, three of these formulations become equivalent at the level of their linear programming relaxations, leaving essentially two distinct integer formulations, commonly denoted by IP1 and IP2. Since the benchmark experiments in [9] did not report meaningful differences between IP1 and IP2 for the types of instances and objectives relevant to our study, we adopt an IP1-type formulation as it is also the most direct to implement for $K_{m,n}$ in an edge-indexed manner.

We adapt this IP1-type formulation to near-interval edge-colorings. The model minimizes the number of used colors while enforcing (i) a proper edge-coloring and (ii) the near-interval condition via the span constraint $\overline{S}(v, \alpha) - \underline{S}(v, \alpha) \leq d_G(v)$ at every vertex.

Finally, we incorporate symmetry breaking as described in [9] to reduce the color-permutation symmetry inherent in edge-coloring models. Concretely, we enforce that the set of used colors forms an initial segment $\{1,2,\dots,t\}$ by the monotonicity constraints $y_c \geq y_{c+1}$, which prunes equivalent solutions without changing feasibility or optimality.

Let $G = K_{m,n}$ with bipartition $V(G) = X \cup Y: X = \{x_1, x_2, \dots, x_m\}, Y = \{y_1, y_2, \dots, y_n\}$. Also let $C = \{1,2,\dots,C_{max}\}$ be the set of available colors. It is known that $w^1(K_{m,n}) \leq n + m - gcd(n,m)$; therefore, $C_{max} = n + m - gcd(n,m)$. We will denote the coloring derived from the optimal solution of the model described below as α. We introduce the following variables:

- $\forall u \in X, v \in Y: x_{uvc} \in \{0,1\}$: 1 if edge (u,v) is colored with color c, 0 otherwise.
- $y_c \in \{0,1\}$: 1 if color c is used in the coloring, 0 otherwise.
- $\forall u \in X: zL_{u,c} \in \{0,1\}$: 1 if color c is used on some edge incident to vertex $u \in X$.
- $\forall v \in Y: zR_{v,c} \in \{0,1\}$: 1 if color c is used on some edge incident to vertex $v \in Y$.
- $\forall u \in X: minL_u \in \{1,\dots,C_{max}\}$: Minimum color used at vertex $u \in X$.
- $\forall u \in X: maxL_u \in \{1,\dots,C_{max}\}$: Maximum color used at vertex $u \in X$.
- $\forall v \in Y: minR_v \in \{1,\dots,C_{max}\}$: Minimum color used at vertex $v \in Y$.
- $\forall v \in Y: maxR_v \in \{1,\dots,C_{max}\}$: Maximum color used at vertex $v \in Y$.

We add a constraint to ensure that each edge receives exactly one color:

$$\forall (u,v) \in E: \quad \sum_{c \in C} x_{uvc} = 1$$

We add the following constraints to ensure the proper edge coloring:

$$\forall u \in X,\ \forall c \in C: \quad \sum_{v \in Y} x_{uvc} \leq 1$$

$$\forall v \in Y,\ \forall c \in C: \quad \sum_{u \in X} x_{uvc} \leq 1$$

We add a constraint to ensure that if any edge is colored with color c, then $y_c = 1$:

$$\forall c \in C: \quad \sum_{(u,v) \in E(G)} x_{uvc} \leq y_c \cdot |E(G)|$$

We add a constraint to ensure that if color $c + 1$ is used, then color c must also be used:

$$\forall c \in \{1,\dots,C_{max} - 1\}: \quad y_c \geq y_{c+1}$$

We add the following constraints to make sure $zL_{u,c}$ and $zR_{v,c}$ variables are correct:

$$\forall u \in X,\ \forall c \in C: \quad zL_{u,c} = \sum_{v \in Y} x_{uvc}$$
$$\forall v \in Y,\ \forall c \in C: \quad zR_{v,c} = \sum_{u \in X} x_{uvc}$$

We add the following constraints to ensure that $minL_u$ and $minR_v$ variables are not greater than the minimum colors $\underline{S}(u, \alpha)$ and $\underline{S}(v, \alpha)$, respectively.

$$\forall u \in X,\ \forall c \in C: \quad minL_u \leq c + C_{max}(1 - zL_{u,c})$$
$$\forall v \in Y,\ \forall c \in C: \quad minR_v \leq c + C_{max}(1 - zR_{v,c})$$

We add the following constraints to ensure that $maxL_u$ and $maxR_v$ variables are not less than the maximum colors $\overline{S}(u, \alpha)$ and $\overline{S}(v, \alpha)$, respectively.

$$\forall u \in X,\ \forall c \in C: \quad maxL_u \geq c - C_{max}(1 - zL_{u,c})$$
$$\forall v \in Y,\ \forall c \in C: \quad maxR_v \geq c - C_{max}(1 - zR_{v,c})$$

We add two more constraints to make sure the resulting coloring will be a near-interval coloring:

$$\forall u \in X: \quad maxL_u - minL_u \leq d_G(u)$$
$$\forall v \in Y: \quad maxR_v - minR_v \leq d_G(v)$$

And finally, we add the objective to minimize the total number of colors used:

$$min \sum_{c \in C} y_c$$

It is easy to see that any feasible solution in the search space to the model described above will correspond to a near-interval coloring, and the optimal one will result in a near-interval t coloring, where $t = w^1(K_{m,n})$. All variables and constraints are implemented using Gurobi's Python API and can be verified in the source code at our github repository.

Main results

Recently Petrosyan and Tsirunyan obtained the following results.

Theorem 1. [8] For any $n, k, c \in \mathbb{N}$ with $c \leq k$, we have

$$w^1(K_{n,(n+1)k-c}) = (n+1)k - c.$$

Corollary 1. [8] For any $n \in \mathbb{N}$, for any $m \in \mathbb{N}$ with $n^2 \leq m$, we have

$$w^1(K_{m,n}) = m.$$

Proposition 1. [8] For any $m, n \in \mathbb{N}$, we have

$$w^1(K_{m,n}) \leq m + n + \min\{-gcd(m,n), 1 - gcd(m+1,n), 1 - gcd(m,n+1), 2 - gcd(m+1,n+1)\}.$$

Corollary 2. [8] For any $n, m \in \mathbb{N}$ with $m \leq n$ and $n + 1 \equiv 0\ (mod\ m+1)$, we have

$$n \leq w^1(K_{m,n}) \leq n + 1.$$

Using the integer linear programming model described above, we were able to obtain the following result.

Theorem 2. For any $n, m \in \mathbb{N}$ with $m \le n$, $max(n, m) \le 20$ and $n + 1 \equiv 0 \ (mod\ m + 1)$, we have $w^1(K_{m,n}) = n$.

Proof. For the cases $n = m$ and $m = 1$ the proof is trivial as $w(K_{n,n}) = w(K_{1,n}) = n$. In case $m = 2$, there are no pairs $(2, n)$ such that $n \le m^2$ and $n + 1 \equiv 0 \ (mod\ 3)$. Therefore, using corollary 2 we can deduce that for any $w^1(K_{2,n}) = n$ where $n \equiv 2 \ (mod\ 3)$.

To complete the proof, it is sufficient to prove the statement above for pairs $(3,7)$, $(3,11)$, $(3,15)$, $(3,19)$, $(4,9)$, $(4,14)$, $(4,19)$, $(5,11)$, $(5,17)$, $(6,13)$, (6,20), (7,15), (8,17), (9,19). You can find all colorings for the given complete bipartite graphs below.

Table 1. The near-interval 7-coloring of $K_{3,7}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x_1	2	1	7	4	5	3	6
x_2	4	2	6	5	7	1	3
x_3	1	3	5	7	6	2	4

Table 2. The near-interval 11-coloring of $K_{3,11}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
x_1	8	1	4	5	10	7	2	6	3	9	11
x_2	6	2	1	4	9	8	3	7	5	11	10
x_3	5	3	2	7	11	10	1	4	6	8	9

Table 3. The near-interval 15-coloring of $K_{3,15}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
x_1	1	9	13	15	11	8	7	2	14	10	12	5	6	3	4
x_2	2	10	15	14	9	11	6	4	12	8	13	3	7	1	5
x_3	3	12	14	13	8	10	9	1	15	7	11	6	5	4	2

Table 4. The near-interval 19-coloring of $K_{3,19}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}
x_1	17	15	4	6	9	10	3	5	11	14	16	18	2	12	19	13	8	1	7
x_2	18	17	5	7	12	9	2	3	10	15	13	19	1	11	16	14	6	4	8
x_3	19	16	3	8	10	7	1	6	13	12	15	17	4	14	18	11	9	2	5

Table 5. The near-interval 9-coloring of $K_{4,9}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	6	4	5	7	9	2	1	8	3
x_2	8	1	6	9	7	3	5	4	2
x_3	9	3	7	6	8	1	2	5	4
x_4	7	2	9	8	5	4	3	6	1

Table 6. The near-interval 14-coloring of $K_{4,14}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}
x_1	10	12	9	4	7	13	5	14	2	8	3	6	1	11
x_2	8	11	5	2	10	14	9	12	3	6	1	7	4	13
x_3	6	13	7	5	9	12	8	11	1	10	4	3	2	14
x_4	9	14	8	1	11	10	6	13	5	7	2	4	3	12

Table 7. The near-interval 19-coloring of $K_{4,19}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}
x_1	6	1	18	19	12	5	10	17	7	9	15	16	11	13	2	4	14	3	8
x_2	4	5	16	15	10	2	11	19	9	8	17	18	12	14	3	1	13	7	6
x_3	8	3	14	18	9	1	12	15	6	11	19	17	13	16	4	2	10	5	7
x_4	5	2	15	17	13	4	14	18	8	7	16	19	10	12	1	3	11	6	9

Table 8. The near-interval 11-coloring of $K_{5,11}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
x_1	5	1	4	9	3	8	10	11	6	7	2
x_2	3	4	2	8	1	10	11	9	7	6	5
x_3	1	3	5	10	2	7	6	8	9	11	4
x_4	6	2	3	7	4	11	9	10	5	8	1
x_5	2	6	1	11	5	9	8	7	4	10	3

Table 9. The near-interval 17-coloring of $K_{5,17}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}
x_1	7	16	15	6	11	9	17	14	2	5	4	10	12	13	8	3	1
x_2	6	11	17	10	12	7	15	13	1	3	2	8	14	16	9	5	4
x_3	4	13	12	7	9	8	14	16	5	1	3	11	15	17	10	2	6
x_4	9	12	14	8	7	10	13	17	4	2	5	6	16	15	11	1	3
x_5	8	14	13	11	10	5	16	15	3	6	1	9	17	12	7	4	2

Table 10. The near-interval 13-coloring of $K_{6,13}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}
x_1	4	3	8	1	9	5	11	7	2	12	13	10	6
x_2	1	7	9	5	13	2	10	6	4	8	11	12	3
x_3	7	6	12	4	11	1	8	5	3	9	10	13	2
x_4	5	4	10	7	12	3	13	2	6	11	9	8	1
x_5	2	1	13	3	8	6	9	4	5	10	12	11	7
x_6	6	5	11	2	10	7	12	3	1	13	8	9	4

Table 11. The near-interval 20-coloring of $K_{6,20}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}	y_{20}
x_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x_2	3	5	1	8	6	4	2	12	11	13	7	10	9	15	16	20	18	14	17	19
x_3	7	1	5	9	2	3	4	10	14	12	6	13	8	17	11	15	16	19	20	18
x_4	4	3	7	5	1	2	6	9	13	15	10	8	11	18	12	17	19	20	14	16
x_5	2	6	4	7	3	1	5	13	8	16	9	11	12	20	10	19	14	15	18	17
x_6	6	7	2	3	4	5	1	14	12	11	8	9	10	19	13	18	20	17	16	15

Table 12. The near-interval 15-coloring of $K_{7,15}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
x_1	7	8	2	13	3	5	12	14	11	6	4	15	10	9	1
x_2	2	3	6	14	5	1	11	12	13	4	7	9	15	10	8
x_3	4	1	3	10	2	6	14	9	15	7	8	12	13	11	5
x_4	8	7	5	9	4	2	10	13	14	1	6	11	12	15	3
x_5	3	4	1	12	6	8	15	10	9	5	2	14	11	13	7
x_6	6	5	8	15	7	3	13	11	12	2	1	10	9	14	4
x_7	5	6	4	11	1	7	9	15	10	8	3	13	14	12	2

Table 13. The near-interval 17-coloring of $K_{8,17}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}
x_1	2	9	3	7	5	1	8	4	14	16	10	11	13	15	12	17	6
x_2	7	2	4	6	8	5	3	1	15	14	13	17	16	12	9	11	10
x_3	9	5	2	1	6	3	4	7	12	17	15	13	10	16	11	14	8
x_4	3	4	8	2	1	7	5	6	11	10	14	12	17	9	15	16	13
x_5	4	1	5	8	2	6	7	3	16	9	12	14	15	17	10	13	11
x_6	5	3	6	4	7	2	1	8	13	11	17	15	9	10	16	12	14
x_7	6	7	1	3	4	8	2	5	17	15	16	10	14	11	13	9	12
x_8	1	8	9	5	3	4	6	2	10	13	11	16	12	14	17	15	7

Table 14. The near-interval 19-coloring of $K_{9,19}$.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}
x_1	3	14	12	7	8	17	1	4	10	11	2	16	6	18	9	19	5	15	13
x_2	5	19	16	11	3	14	6	2	8	17	9	12	1	13	4	10	7	18	15
x_3	1	18	15	4	2	12	10	3	5	14	6	11	7	19	8	13	9	16	17
x_4	2	17	11	6	7	15	9	5	1	13	4	14	10	16	3	12	8	19	18
x_5	9	11	14	5	6	19	3	10	2	16	8	18	4	17	7	15	1	13	12
x_6	6	15	19	9	10	11	7	8	4	18	5	13	2	12	1	16	3	17	14
x_7	7	13	17	2	5	16	4	9	3	12	1	15	8	11	10	18	6	14	19
x_8	8	12	13	10	4	18	5	1	9	19	7	17	3	15	6	14	2	11	16
x_9	4	16	18	8	1	13	2	6	7	15	3	19	9	14	5	17	10	12	11

□

Based on our computer experiments, we strongly believe that the following conjecture is true.

Conjecture 1. For any $n, m \in \mathbb{N}$ with $m \leq n$ and $n + 1 \equiv 0 \ (mod\ m + 1)$, we have $w^1(K_{m,n}) = n$.

As a result of our computer experiments we have found the exact values of $K_{n,m}$ for all $n, m \in \mathbb{N}$ with $n, m \leq 20$.

Table 15. Exact values of $w^1(K_{n,m})$ with $n, m \leq 20$.

n/m	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	3	3	3	4	6	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4	4	4	4	4	5	7	8	8	9	10	12	12	13	14	15	16	17	18	19	20
5	5	5	6	5	5	6	9	10	10	10	11	12	14	15	15	16	17	18	20	20
6	6	6	6	7	6	6	7	10	11	12	12	12	13	14	16	17	18	18	19	20
7	7	7	7	8	9	7	7	8	12	13	14	15	14	14	15	16	19	20	21	21
8	8	8	8	8	10	10	8	8	9	13	15	15	17	17	16	16	17	18	21	22
9	9	9	9	9	10	11	12	9	9	10	15	16	17	18	19	20	18	18	19	20
10	10	10	10	10	10	12	13	13	10	10	11	16	18	19	19	21	22	22	20	20
11	11	11	11	12	11	12	14	15	15	11	11	12	18	20	21	21	22	24	25	25
12	12	12	12	12	12	12	15	15	16	16	12	12	13	19	21	22	23	23	25	26
13	13	13	13	13	14	13	14	17	17	18	18	13	13	14	21	23	24	25	25	26
14	14	14	14	14	15	14	14	17	18	19	20	19	14	14	15	22	25	26	27	27
15	15	15	15	15	15	16	15	16	19	19	21	21	21	15	15	16	24	26	28	28
16	16	16	16	16	16	17	16	16	20	21	21	22	23	22	16	16	17	25	28	29
17	17	17	17	17	17	18	19	17	18	22	22	23	24	25	24	17	17	18	27	30
18	18	18	18	18	18	18	20	18	18	22	24	23	25	26	26	25	18	18	19	28
19	19	19	19	19	20	19	21	21	19	20	25	25	25	27	28	28	27	19	19	20
20	20	20	20	20	20	20	21	22	20	20	25	26	26	27	28	29	30	28	20	20

Observation. For all $n, m \in \mathbb{N}$ with $n, m \leq 20$ we have the following: $\max_{n,m \leq 20} \left(\frac{w^1(K_{n,m})}{\max(n,m)} \right) = 1.5.$

Proof. It is easy to see from Table 1 that this value is correct and was achieved using $n = 17, m = 20$ or $n = 20, m = 17$ □

Conclusion

This work is devoted to the study of near-interval colorings of complete bipartite graphs. In particular, new exact values of $w^1(K_{m,n})$ were obtained for some complete bipartite graphs.

The main results obtained in this work are the following:

For any $n, m \in \mathbb{N}$ with $m \leq n$, $\max(n, m) \leq 20$ and $n + 1 \equiv 0 \ (mod\ m + 1)$, we have

$$w^1(K_{m,n}) = n.$$

For all $n, m \in \mathbb{N}$ with $n, m \leq 20$ we have the following:

$$\max_{n,m \leq 20} \left(w^1(K_{n,m}) / \max(n, m) \right) = 1.5$$

We also formulate the following conjectures:

For any $n, m \in \mathbb{N}$ with $m \leq n$ and $n + 1 \equiv 0 \ (mod\ m + 1)$, we have

$$w^1(K_{m,n}) = n.$$

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МОДЕЛЬ ЦЕЛОЧИСЛЕННОГО ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ ДЛЯ ИССЛЕДОВАНИЯ ПОЧТИ-ИНТЕРВАЛЬНЫХ РЕБЕРНЫХ РАСКРАСОК

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АННОТАЦИЯ

Правильной реберной раскраской графа G называется отображение $\alpha: E(G) \to \mathbb{N}$, такое что $\alpha(e) \neq \alpha(e')$ для любой пары смежных ребер e и e' графа G. Если α-правильная реберная раскраска графа G и $v \in V(G)$, то через $S_G(v,\alpha)$ обозначается множество цветов, встречающихся на ребрах, инцидентных вершине v.

Правильная реберная раскраска α графа G цветами $1,\dots,t$ называется «почти-интервальной t-раскраской», если используются все цвета и для каждой вершины $v \in V(G)$ множество $S_G(v,\alpha)$ является интервалом целых чисел, содержащим не более одного разрыва. Если граф G допускает такую раскраску, то минимальное число цветов в почти-интервальной раскраске графа G обозначается через $w^1(G)$. Известно, что все полные двудольные графы допускают почти-интервальные раскраски.

В данной работе предлагается модель целочисленного линейного программирования для определения или оценки параметра $w^1(K_{m,n})$ $(m,n \in \mathbb{N})$ для полных двудольных графов.

Ключевые слова: правильная реберная раскраска, почти-интервальная раскраска, интервальная раскраска, полный двудольный граф, целочисленное линейное программирование.